Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Tuesday 6 June 2023

Afternoon (Time: 2 hours)

Paper reference **9MA0/01**

Mathematics

Advanced

PAPER 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,dx$$

writing each term in simplest form.

(4)

$\frac{1}{3}\int x^{\frac{1}{2}}(2x-5)\,dx$ take $\frac{1}{3}$ out infront of integral

$= \frac{1}{3}\int 2x^{\frac{3}{2}} - 5x^{\frac{1}{2}}\,dx$ M1A1 Expand the bracket & use Index Rule: $x^a \times x^b = x^{ab}$.

$= \frac{1}{3}\left(\frac{2}{\frac{5}{2}}x^{\frac{5}{2}} - \frac{5}{\frac{3}{2}}x^{\frac{3}{2}}\right)$ ★ Simple Integration: $\int x^n\,dx = \frac{1}{n}x^{n+1} + c$

$= \frac{1}{3}\left(\frac{4}{5}x^{\frac{5}{2}} - \frac{10}{3}x^{\frac{3}{2}}\right) + c$ dM1

$= \frac{4}{15}x^{\frac{5}{2}} - \frac{10}{9}x^{\frac{3}{2}} + c$ A1

Question 1 continued

(Total for Question 1 is 4 marks)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \qquad x \in \mathbb{R}$$

where a is a positive constant.

Given $(x - a)$ is a factor of $f(x)$,

(a) show that

$$a(4a^2 + 5a - 6) = 0$$

(2)

(b) Hence

(i) find the value of a

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 3$$

(4)

(a) Since $(x-a)$ is a factor, $f(a) = 0$ must be true

$x = a$

We can hence substitute $x = a$ into the equation:

$f(a) = 0 = 4(a)^3 + 5(a)^2 - 10(a) + 4a$ M1

$0 = 4a^3 + 5a^2 - 10a + 4a$ collect like terms

$0 = 4a^3 + 5a^2 - 6a$ factorize

$0 = a(4a^2 + 5a - 6)$ A1

Hence Shown

(b) i. $0 = a(4a^2 + 5a - 6)$ — factorize → $0 = a(4a-3)(a+2)$

$\therefore a = 0,\ a = -2,\ a = \frac{3}{4}$ B1

Reject as a is a positive constant.

ii. $f(x) = 3$ and substitute $a = \frac{3}{4}$

$f(x) = 4x^3 + 5x^2 - 10x + \cancel{4}\left(\frac{3}{\cancel{4}}\right)$ substitute $a = \frac{3}{4}$

$f(x) = 4x^3 + 5x^2 - 10x + 3 = 3$ M1 $f(x) = 3$

$4x^3 + 5x^2 - 10x = 0$

$x = 0 \leftarrow x(4x^2 + 5x - 10) = 0$

Use Quadratic Formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-5 \pm \sqrt{185}}{8},\ x = 0$ B1A1

P 7 2 8 0 4 A 0 4 4 4

Question 2 continued

(Total for Question 2 is 6 marks)

3. Relative to a fixed origin O

- the point A has position vector $5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- the point B has position vector $2\mathbf{i} + 4\mathbf{j} + a\mathbf{k}$

where a is a positive integer.

(a) Show that $|\overrightarrow{OA}| = \sqrt{38}$

(1)

(b) Find the smallest value of a for which

$$|\overrightarrow{OB}| > |\overrightarrow{OA}|$$

(2)

Let's first write $\overrightarrow{OA}$ and $\overrightarrow{OB}$ as column vectors:

$\overrightarrow{OA} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$

$\overrightarrow{OB} = \begin{pmatrix} 2 \\ 4 \\ a \end{pmatrix}$

(a) $|\overrightarrow{OA}|$ is the magnitude of $\overrightarrow{OA}$ which we get by using Pythagoras' Theorem:

Use $\sqrt{a^2+b^2+c^2}$:

$|\overrightarrow{OA}| = \sqrt{5^2+3^2+2^2}$

$= \sqrt{25+9+4} = \sqrt{38}$ hence shown B1

(b) Let's find $|\overrightarrow{OB}|$ using Pythagoras' Theorem:

Use $\sqrt{a^2+b^2+c^2}$:

$|\overrightarrow{OB}| = \sqrt{2^2+4^2+a^2}$

$= \sqrt{4+16+a^2} = \sqrt{20+a^2}$

Since a is an integer:

for $20 + a^2 > 38$,

$a^2 > 18$.

The first square number larger than 18 is $25\ (5^2)$ M1

$\therefore$ smallest a-value: $\boxed{a = 5}$ A1

Question 3 continued

(Total for Question 3 is 3 marks)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$ where $x \in \mathbb{R}$

Given that

- $f'(x) = 2x + \frac{1}{2}\cos x$
- the curve has a stationary point with x coordinate α
- α is small $\longrightarrow$ small angle approx.

(a) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

The point $P(0, 3)$ lies on C

(b) Find the equation of the tangent to the curve at P, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

(a) At the stationary point $x = a$, $f'(x) = 0$.

Substitute:

small angle approx. for cos: $\cos\theta \approx 1 - \frac{\theta^2}{2}$

$$0 = 2a + \frac{1}{2}\left(1 - \frac{a^2}{2}\right) \quad \text{M1 Expand}$$

$$0 = 2a + \frac{1}{2} - \frac{a^2}{4}$$

$$0 = \frac{a^2}{4} - 2a - \frac{1}{2}$$

$\times 4$

$$0 = a^2 - 8a - 2 \quad \text{Use Quadratic Formula: } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 4 \pm 3\sqrt{2} \quad \text{dM1}$$

However a must be small $\therefore$ reject $4 + 3\sqrt{2}$.

$$\therefore a = -0.243 \text{ to 3dp} \quad \text{A1}$$

(b) Substitute $x = 0$ into $f'(x)$ to get the gradient

$$f'(0) = 2(0) + \frac{1}{2}\cos(0)$$

$$m = \frac{1}{2} \quad \text{M1}$$

Use $y - y_1 = m(x - x_1)$ with $m = \frac{1}{2}$ and $(0, 3)$

$$y - 3 = \frac{1}{2}(x - 0)$$

$$y = \frac{1}{2}x + 3 \quad \text{A1}$$

Question 4 continued

(Total for Question 4 is 5 marks)

5. A continuous curve has equation $y = f(x)$.

The table shows corresponding values of x and y for this curve, where a and b are constants.

x	3	3.2	3.4	3.6	3.8	4
y	a	16.8	b	20.2	18.7	13.5

The trapezium rule is used, with all the y values in the table, to find an approximate area under the curve between $x = 3$ and $x = 4$

Given that this area is 17.59

(a) show that $a + 2b = 51$

(3)

Given also that the sum of all the y values in the table is 97.2

(b) find the value of a and the value of b

(3)

(a) From the formula booklet:

$A = \frac{1}{2}h(y_0 + y_n + 2(y_1 + y_2 + y_3 \dots + y_{n-1}))$ ← trapezium rule

Substitute:

$17.59 = \frac{1}{2}(\underbrace{0.2}_{h})(a + 13.5 + 2(16.8 + b + 20.2 + 18.7))$ M1 B1

$\frac{17.59}{0.1} = a + 13.5 + 2(55.7) + 2b$

$175.9 = 124.9 + a + 2b$

$\therefore \quad 51 = a + 2b$ hence shown A1

(b) Σy values $= 97.2 \longrightarrow 97.2 = a + b + 16.8 + 20.2 + 18.7 + 13.5$ M1

$97.2 = a + b + 69.2$

$28 = a + b$

Solve Simultaneously:

$a = 28 - b$

$51 = (28 - b) + 2b$

$51 - 28 = b \longrightarrow b = 23$

$a = 28 - 23 = 5$

$\therefore \; a = 5, \; b = 23$ A1A1

Question 5 continued

(Total for Question 5 is 6 marks)

P 7 2 8 0 4 A 0 1 1 4 4

6. $a = \log_2 x \qquad b = \log_2(x + 8)$

Express in terms of a and/or b

(a) $\log_2 \sqrt{x}$ **(1)**

(b) $\log_2(x^2 + 8x)$ **(2)**

(c) $\log_2\left(8 + \dfrac{64}{x}\right)$

Give your answer in simplest form. **(3)**

(a) $\log_2 \sqrt{x} = \log_2 x^{\frac{1}{2}}$

$= \frac{1}{2}\log_2 x$

$= \frac{1}{2}a$ B1

Log Rule Used

$\log x^a = a \log x$

(b) $\log_2(x^2 + 8x) = \log_2(x(x+8))$ factorize

$= \log_2 x + \log_2(x+8)$ M1 $\log(ab) = \log a + \log b$

$= a + b$ A1

It would be an easy mistake to rewrite this as $\log_2 x^2 + \log_2 8x$!

(c) $\log_2\left(8 + \frac{64}{x}\right) = \log_2\left(\frac{8}{x}(x+8)\right)$ B1 factorize $\div \frac{8}{x}$

$= \log_2 \frac{8}{x} + \log_2(x+8)$ $\log(ab) = \log a + \log b$

$= \log_2 8 + \log_2 x^{-1} + \log_2(x+8)$ $\log(ab) = \log a + \log b$ (again)

M1 $= 3 - \log_2 x + \log_2(x+8)$ $\log x^a = a \log x$

$= 3 - a + b$ A1

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Question 6 continued

(Total for Question 6 is 6 marks)

7. The function f is defined by

$$f(x) = 3 + \sqrt{x-2} \qquad x \in \mathbb{R} \quad x > 2$$

(a) State the range of f **(1)**

(b) Find f^{-1} **(3)**

The function g is defined by

$$g(x) = \frac{15}{x-3} \qquad x \in \mathbb{R} \quad x \neq 3$$

(c) Find gf(6) **(2)**

(d) Find the exact value of the constant a for which

$$f(a^2 + 2) = g(a)$$

(2)

(a) Substitute in the domain $x > 2$:

$f(2) = 3 + \sqrt{0}$

$f(2) = 3$

$\therefore f(x) > 3$ range B1

(b) First set $f(x) = y$

$y = 3 + \sqrt{x-2}$

Apply $y = x$ and $x = y$

$x = 3 + \sqrt{y-2}$ M1

Make y the subject

$x - 3 = \sqrt{y-2}$

$(x-3)^2 = y - 2$

$y = (x-3)^2 + 2$

Set $y = f^{-1}(x)$

$f^{-1}(x) = (x-3)^2 + 2$, $x > 3$ $x \in \mathbb{R}$ B1 Remember to always state the domain, it gets a mark! A1

(c) $f(6) = 3 + \sqrt{6-2} = 5$ ← this output becomes the input for g(x)

$g(5) = \frac{15}{5-3} = \frac{15}{2}$ M1

$\therefore gf(6) = \frac{15}{2}$ A1

Question 7 continued

(d) $f(a^2+2) = 3+\sqrt{a^2+2-2} = 3+\sqrt{a^2} = 3+a$ Substitute $x=a^2+2$

$g(a) = \dfrac{15}{a-3}$

Equate these:

$$3+a = \frac{15}{a-3}$$

★ this is difference of two squares format! ← $(a-3)(a+3) = 15$ Expand

$a^2-9=15$ M1

$a^2=24$

$a=\pm 2\sqrt{6}$

for $-2\sqrt{6}$: $f(a^2+2) \neq g(a)$ ∴ Reject.

$a = 2\sqrt{6}$ A1

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8.

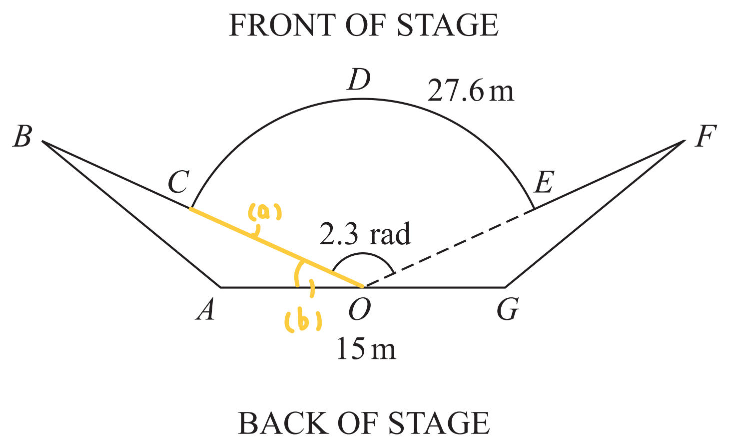

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles ABO and GFO joined to a sector $OCDEO$ of a circle, centre O, where

- angle $COE = 2.3$ radians
- arc length $CDE = 27.6$ m
- AOG is a straight line of length 15 m

(a) Show that $OC = 12$ m.

(2)

(b) Show that the size of angle AOB is 0.421 radians correct to 3 decimal places.

(2)

Given that the total length of the front of the stage, $BCDEF$, is 35 m,

(c) find the total area of the stage, giving your answer to the nearest square metre.

(6)

(a) We need to find OC, the radius of the circle center O.

Use arc length $= r\theta$:

$27.6 = 2.3 \times r$ M1

$r = \frac{27.6}{2.3} = 12$ A1 hence shown

(b) Since AOG is a straight line:

$A\hat{O}B + G\hat{O}F + C\hat{O}E = \pi$ rad.

since ΔABO and ΔGFO are congruent, $A\hat{O}B + G\hat{O}F$.

$\therefore 2A\hat{O}B + 2.3 = \pi$ M1

$A\hat{O}B = \frac{\pi - 2.3}{2} = 0.421$ rad hence shown

A1

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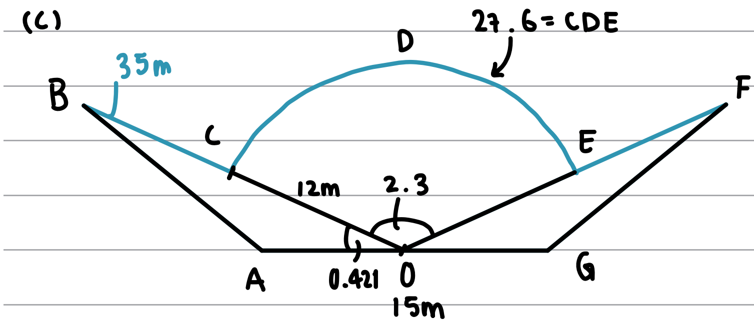
Question 8 continued

(c)



$BC = \frac{35 - 27.6}{2} = 3.7$

$\therefore OB = 12 + 3.7 = 15.7m$ B1

→ Area of each triangle

Use $A = \frac{1}{2}ab\sin C$:

AO   BO = OC + CB

$A = \frac{1}{2}(7.5)(12 + 3.7)\sin(0.421)$ M1

$= 24.0m^2$

→ Sector OCDE

Use $A = \frac{1}{2}r^2\theta$:

$A = \frac{1}{2}(12)^2 \times 2.3$ M1

$= 165.6m^2$ A1

Total Area : $165.6 + 2(24.1)$ dM1

$= 214m^2$ A1

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9. The first three terms of a geometric sequence are

1st 2nd 3rd

$$3k + 4 \qquad 12 - 3k \qquad k + 16$$

where k is a constant.

(a) Show that k satisfies the equation

$$3k^2 - 62k + 40 = 0$$

(2)

Given that the sequence converges,

(b) (i) find the value of k, giving a reason for your answer,

(ii) find the value of S_∞

(5)

(a) Since it's a geometric sequence the ratio between consecutive terms is constant:

$$\frac{2^{nd}}{1^{st}} = \frac{3^{rd}}{2^{nd}}$$

$$\frac{12-3k}{3k+4} = \frac{k+16}{12-3k} \quad \text{M1}$$

$$(12-3k)^2 = (k+16)(3k+4)$$

$$144 - 72k + 9k^2 = 3k^2 + 52k + 64$$

$$0 = 6k^2 - 124k + 80 \quad \div 2$$

$$0 = 3k^2 - 62k + 40 \quad \text{A1}$$

hence shown

(b) i. "converges" → common ratio $|r| < 1$

$$0 = (3k-2)(k-20)$$

$$k = \frac{2}{3} \qquad k = 20 \quad \text{M1}$$

$k = 20$: $64, -48, 36$ $\quad r = -\frac{3}{4}$

$k = \frac{2}{3}$: $6, 10, \frac{50}{3}$ $\quad r = \frac{5}{3}$

$k = 20$ as for $k = 20$, $r = -\frac{3}{4}$ and for series to converge → $|r| < 1$ A1

ii. Formula for sum to infininty:

$$S_\infty = \frac{a}{1-r}$$

$a = 64$, $r = -\frac{3}{4}$ B1

$$S_\infty = \frac{64}{1 - -\frac{3}{4}} = \frac{256}{7} \longrightarrow S_\infty = \frac{256}{7} \quad \text{A1}$$

M1

Question 9 continued

(Total for Question 9 is 7 marks)

10. A circle C has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where k is a constant.

(a) Find in terms of k,

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = 2x - 1$ intersects C at 2 distinct points.

(b) Find the range of possible values of k.

(6)

(a) Complete the square to get the standard circle formula $(x-a)^2 + (y-b)^2 = r^2$

center (a, b)

Radius r

$$x^2 + 6kx + y^2 - 2ky + 7 = 0$$
$$(x+3k)^2 - (3k)^2 + (y-k)^2 - (-k)^2 + 7 = 0$$
$$(x+3k)^2 - 9k^2 + (y-k)^2 - k^2 + 7 = 0$$
$$(x+3k)^2 + (y-k)^2 = 10k^2 - 7 \quad \text{M1}$$

i. $(-3k, k)$ B1

ii. $\sqrt{10k^2-7}$ A1

(b) Substitute $y = 2x - 1$ into $x^2 + y^2 + 6kx - 2ky + 7 = 0$

$$x^2 + (2x-1)^2 + 6kx - 2k(2x-1) + 7 = 0 \quad \text{M1}$$

collect like terms

$$x^2 + 4x^2 - 4x + 1 + 6kx - 4kx + 2k + 7 = 0$$
$$5x^2 - 4x + 2kx + 8 + 2k = 0 \qquad ax^2 + bx + c = 0$$
$$a = 5 \qquad 2k - 4 = b \qquad c = 8 + 2k \quad \text{A1}$$

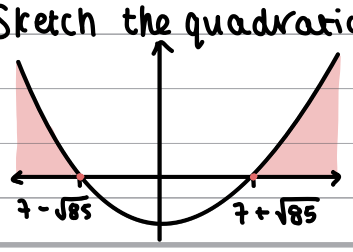
Use discriminant $b^2 - 4ac > 0$ as the equation must have 2 distinct solutions for the line to intersect the circle twice.

$$(2k-4)^2 - 4(5)(8+2k) > 0 \quad \text{dM1}$$
$$4k^2 - 16k + 16 - 20(2k+8) = 0$$
$$4k^2 - 16k + 16 - 40k - 160 = 0$$
$$4k^2 - 56k - 144 = 0$$

Use Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to get k:

$$k = 7 \pm \sqrt{85} \quad \text{A1}$$

Sketch the quadratic:



as we want $b^2 - 4ac > 0$, we want the area above 0.

$\therefore k > 7 + \sqrt{85}, \; k < 7 - \sqrt{85}$

A1 dd M1

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 9 marks)

11.

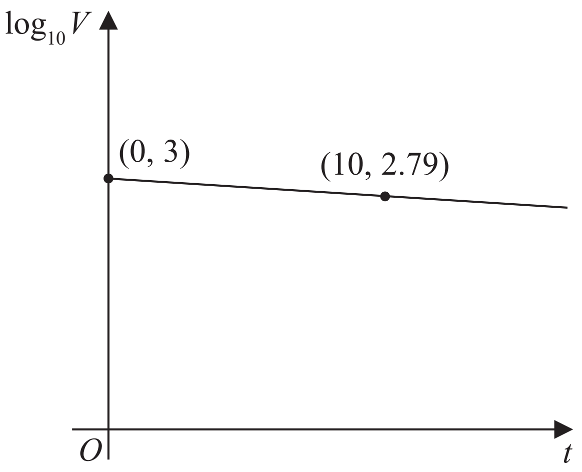


Figure 2

The value, V pounds, of a mobile phone, t months after it was bought, is modelled by

$$V = ab^t$$

where a and b are constants.

Figure 2 shows the linear relationship between $\log_{10} V$ and t.

The line passes through the points (0, 3) and (10, 2.79) as shown.

Using these points,

(a) find the initial value of the phone,

(2)

(b) find a complete equation for V in terms of t, giving the exact value of a and giving the value of b to 3 significant figures.

(3)

Exactly 2 years after it was bought, the value of the phone was £320

(c) Use this information to evaluate the reliability of the model.

(2)

(a) the initial value is at $t=0$:

$\log_{10} V = 3$ at $t = 0$

$10^3 = V$ M1

$V = £1000$ Initial Value A1

(b) use $y - y_1 = m(x - x_1)$

Get $m = \frac{3-2.79}{0-10} = -0.021$

$\log_{10} V - 3 = -0.021(t - 0)$

$\log_{10} V = 3 - 0.021t$ M1

$V = ab^t$

$\log_{10} V = \log_{10} ab^t$

$\log_{10} V = \log_{10} a + t\log_{10} b$

$\log_{10} a = 3$ $\quad a = 1000$

$\log_{10} b = -0.021$ $\quad b = 10^{-0.021}$ M1

$V = ab^t$

$\therefore V = 1000 \times 0.953^t$ A1

P 7 2 8 0 4 A 0 2 8 4 4

Question 11 continued

(c) 2 years → 24 months ∴ $t = 24$ Substitute

$V = 1000 \times 0.953^{24}$

$V = 315$ £ M1

As £315 is close to £320, the model is suitable A1

Question 11 continued

Question 11 continued

(Total for Question 11 is 7 marks)

12.
$$y = \sin x$$

where x is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for $\sin(A \pm B)$
- assume that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

From the formula booklet:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

for this case:

$$y = \sin x$$

expand using $\sin(A+B) = \sin A\cos B + \cos A \sin B$

B1
$$\frac{dy}{dx} = \frac{\sin(x+h) - \sin x}{h}$$

M1A1
$$= \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \frac{\sin x \cos h - \sin x + \cos x \sin h}{h}$$

$$= \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$

$$= \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)$$

As $h \to 0$: $\left(\frac{\cos h - 1}{h}\right) \to 0$ and $\left(\frac{\sin h}{h}\right) \to 1$ dM1

$$\therefore = \sin x(0) + \cos x(1)$$

$$= \cos x = \frac{dy}{dx}$$ hence shown

A1

P 7 2 8 0 4 A 0 3 2 4 4

Question 12 continued

(Total for Question 12 is 5 marks)

13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where a and b are positive constants.

(a) Find a complete equation for the model.
(3)

(b) Use the model to determine the height of the carriage above the ground when $t = 40$
(1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 29\cos(9t + \alpha)° + \beta \qquad 0 \leqslant \alpha < 360°$$

where α and β are constants.

(c) Find a complete equation for the alternative model.
(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.
(1)

(a) $H = a - b(t-20)^2$ completed square form $\therefore$

Maximum $\rightarrow$ $(20, a)$

$\therefore$ $a = 60$ m A1

when $t = 0$, $H = 2$:

$2 = 60 - b(0-20)^2$ solve for b M1

$-58 = -b(400)$

$b = \frac{-58}{-400} = \frac{29}{200} = 0.145$

$\therefore$ $b = 0.145$

complete model: $H = 60 - 0.145(t-20)^2$ A1

Question 13 continued

(b) Substitute $t = 40$:

$$H = 60 - 0.145(40-20)^2$$

$$H = 2m$$ B1

(c) $$H = 29\cos(9t + \alpha) + \beta$$

Maximum $H = 60$, reached when cos is maximum $\cos_{max} = 1$

$$60 = 29(1) + \beta \rightarrow \beta = 31$$

$t = 0$, $H = 2$:

$$2 = 29\cos(\alpha) + 31$$

$$-29 = 29\cos(\alpha)$$

$$\cos\alpha = -1 \rightarrow \alpha = \cos^{-1}(-1)$$

$$\alpha = 180^\circ, \beta = 31$$ M1

complete model: $H = 29\cos(9t + 180) + 31$ A1

(b) The second model allows for the carriage to complete more than one circuit. B1

Question 13 continued

Question 13 continued

(Total for Question 13 is 7 marks)

14. Prove, using algebra, that

$$(n+1)^3 - n^3$$

is odd for all $n \in \mathbb{N}$

(4)

When n is even:

$n = 2k$ → multiple of 2 ∴ even

use the binomial expansion

$(2k+1)^3 - (2k)^3$

$= (2k)^3 + 3(2k)^2(1) + 3(2k)(1)^2 + 1^3 - 8k^3$

$= \cancel{8k^3} + 3(4k^2) + 6k + 1 - \cancel{8k^3}$

$= 12k^2 + 6k + 1$

$= 2(6k^2+3k) + 1$ M1

this number is even since it's a multiple of 2.

Any even number +1 is odd, ∴ when n is even, $(n+1)^3 - n^3$ is odd A1

When n is odd:

$n = 2k+1$

$(2k+1+1)^3 - (2k+1)^3$ use the binomial expansion

use the binomial expansion

$= (2k+2)^3 - (8k^3 + 3(4k^2) + 6k + 1)$

$= (\cancel{8k^3} + 3(2k)^2(2) + 3(2k)(2)^2 + 2^3) - \cancel{8k^3} - 12k^2 - 6k - 1$

$= 24k^2 + 24k + 8 - 12k^2 - 6k - 1$

$= 12k^2 + 18k + 7$

$= 2(6k^2 + 9k) + 7$ dM1

this number is even since it's a multiple of 2.

Any even number +1 is odd, ∴ when n is odd, $(n+1)^3 - n^3$ is odd

Hence, $(n+1)^3 - n^3$ is odd for all $n \in \mathbb{N}$ proven. A1

Question 14 continued

(Total for Question 14 is 4 marks)

15. A curve has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \qquad x > \ln\sqrt[3]{2}$$

(a) Show that

$$\mathrm{f}'(x) = \frac{7e^x(e^{3x}(2 - x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where A and B are constants to be found.

(5)

(b) Hence show that the x coordinates of the turning points of the curve are solutions of the equation

$f'(x) = 0$

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2)

The equation $x = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 on page 42

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β

(1)

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x_2

(ii) the value of β

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

(2)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 15 continued

(a) We are asked to differentiate:

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x}-2}}$$

We see it's a division ∴ we need Quotient rule and the numerator is a multiplication ∴ we need Product rule.

Quotient Rule: $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$

Product Rule: $\frac{d}{dx}(uv) = uv' + vu'$

First Quotient Rule:

$u = 7xe^x$ $\qquad u' =$ ⟶ Product Rule: $a = 7x$ $\quad b' = 7$

$v = \sqrt{e^{3x}-2}$ $\qquad v' = \frac{3}{2}e^{3x}(e^{3x}-2)^{-\frac{1}{2}}$ $\qquad b = e^x$ $\quad a' = e^x$

$u' = 7xe^x + 7e^x$. M1A1

*chain rule: multiply by the derivative of the bracket!

$$\therefore f'(x) = \frac{\overset{u'}{(7xe^x + 7e^x)}\overset{v}{\sqrt{e^{3x}-2}} - \overset{u}{(7xe^x)} \cdot \overset{v'}{\frac{3}{2}e^{3x}(e^{3x}-2)^{-\frac{1}{2}}}}{(\underset{v}{\sqrt{e^{3x}-2}})^2} \quad \text{dM1}$$

$$= \frac{7e^x\left[(x+1)(e^{3x}-2)^{\frac{1}{2}} - \frac{3}{2}xe^{3x}(e^{3x}-2)^{-\frac{1}{2}}\right]}{e^{3x}-2} \quad \text{factor out } 7e^x$$

$$= \frac{7e^x(e^{3x}-2)^{-\frac{1}{2}}\left[(x+1)(e^{3x}-2) - \frac{3}{2}xe^{3x}\right]}{(e^{3x}-2)} \quad \text{factor out } (e^{3x}-2)^{-\frac{1}{2}}$$

$$= \frac{7e^x\left[2(x+1)(e^{3x}-2) - 3xe^{3x}\right]}{2(e^{3x}-2)\times(e^{3x}-2)^{\frac{1}{2}}} \quad \tfrac{1}{2}(e^{3x}-2)^{-\frac{1}{2}} \text{ to denominator}$$

— index rule: $x^a \times x^b = x^{a+b}$

$$= \frac{7e^x(2(xe^{3x} - 2x + e^{3x} - 2) - 3xe^{3x})}{2(e^{3x}-2)^{\frac{3}{2}}} \quad \text{expand numerator}$$

$$= \frac{7e^x(2xe^{3x} - 4x + 2e^{3x} - 4 - 3xe^{3x})}{2(e^{3x}-2)^{\frac{3}{2}}} \quad \text{expand numerator}$$

A1 $$= \frac{7e^x(e^{3x}(2-x) - 4x - 4)}{2(e^{3x}-2)^{\frac{3}{2}}} \quad \text{collect like terms}$$

as required

Question 15 continued

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

(c)

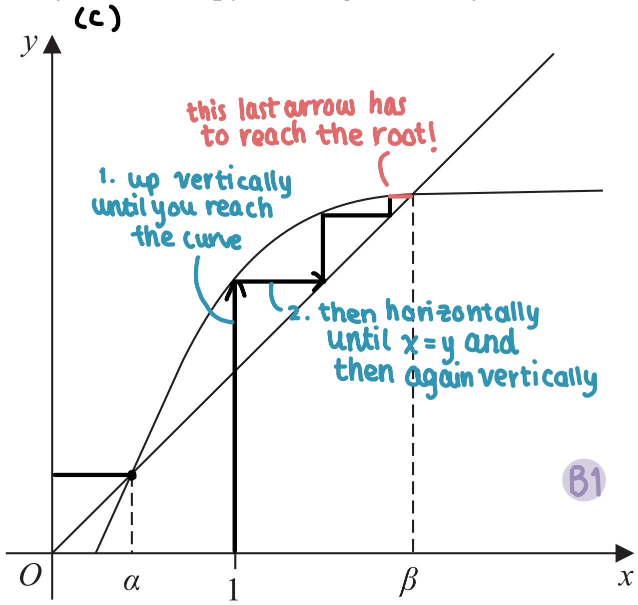
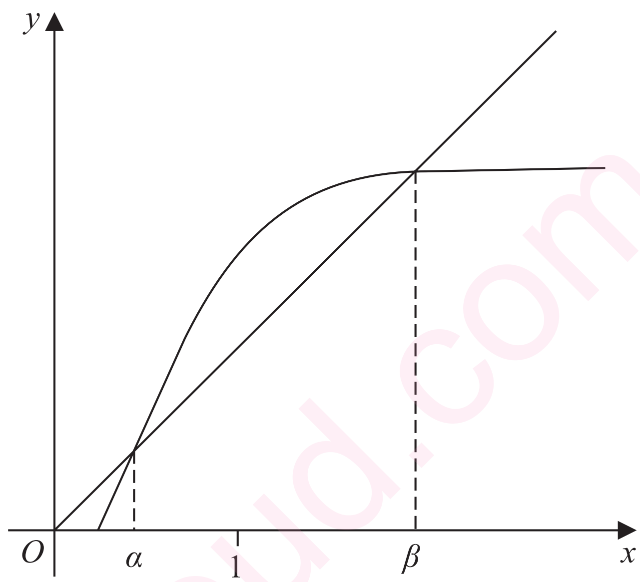


Diagram 1



copy of Diagram 1

(b) The turning points occur when $f'(x)=0$:

$$\frac{7e^{x}(e^{3x}(2-x)-4x-4)}{2(e^{3x}-2)^{\frac{3}{2}}}=0$$

$$e^{3x}(2-x)-4x-4=0 \quad \text{M1}$$

$$2e^{3x}-xe^{3x}-4x-4=0$$

$$xe^{3x}+4x=2e^{3x}-4$$

$$x(e^{3x}+4)=2e^{3x}-4$$

A1 $\quad x=\dfrac{2e^{3x}-4}{e^{3x}+4}$ as required

(d) i. substitute $x_1=1$:

$$x_2=\frac{2e^{3(1)}-4}{e^{3(1)}+4}=\frac{2e^{3}-4}{e^{3}+4} \quad \text{M1}$$

$= 1.5017756 \longrightarrow 1.502$ to 4s.f. A1

ii. $\beta = 1.968$ dB1

Question 15 continued

(e) We will use the function $g(x) = \dfrac{2e^{3x} - 4}{e^{3x} + 4} - x$ and we will substitute 0.4315 and 0.4325. M1

Why? We want values slightly above and slightly below 0.432 ⟶

0.431 0.432 0.433

0.4315 0.4325

$g(0.4315) = -0.000297$

$g(0.4325) = 0.000947$

Since there is a change of sign in this interval of the continuous function $f'(x)$, we can conclude that $\alpha = 0.432$. A1

Question 15 continued

(Total for Question 15 is 13 marks)

TOTAL FOR PAPER IS 100 MARKS